Internship or Schooling?

Based on Ben-Porath Human Capital (HC) Model

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Setting

- Consider individual human capital accumulation (HC acc), using time as inputs. The wage rate is constant.
- The individual has a fixed (flow) endowment of 1 unit of time for market activities, and life time horizon T is fixed.
- The individual decides between schooling and internship & between HC acc and work.
- Key assumptions:
  1. No preference about how time is divided between market activities.
  2. Perfect capital markets: unlimited ability to borrow and lend. Hence her goal is to maximize the PDV of lifetime (net) income.
- Internships offer industry-specific experience, and education offers general knowledge. Firms are heterogeneous while individuals are homogeneous.

Notations

- $[0, T]$: time horizon
- $K_g(t)$: General HC (accumulated by education)
- $K_i(t)$: Industry specific HC (accumulated by internships)
- $K(t)$: Total HC, a combination of both types, calculated as $K(t) = \gamma_1 K_g(t) + \gamma_2 K_i(t)$, where γ_1 and γ_2 are weights that reflect the importance of general and industry-specific knowledge in determining wages.
- $s(t)$: share of (unit) time endowment devoted to HC acc at t
- p_1, p_2: price of education and of internship (can be negative for income)
- $r \geq 0$: constant interest rate

The (one) state variable is K, the (two) controls are t_1, t_2.

Different Firms/Industries

Different industries or firms value industry-specific human capital differently. For industry j:

$$K_i^j(t) = \text{industry-specific experience for industry } j.$$

The wage in industry j depends on both general human capital and the specific experience in that industry:

$$w^j(t) = w_0^j \left(\gamma_1 K_g(t) + \gamma_2 K_i^j(t) \right),$$

where w_0^j represents the base wage rate in industry j, which could vary across industries.

And earnings are product of wage and time working and "effectiveness":

$$E = w(1-s)K$$

Law of Motion (LOM) for HC

General Knowledge (Accumulated via Education):

$$\dot{K}_g(t) = \beta_g t_2 K_g(t),$$

where t_2 represents the time spent on education, and β_g represents the productivity of education in increasing general human capital.
Industry-Specific Experience (Accumulated via Internships):

$$\dot{K}_i^j(t) = \beta_i^j t_1^j K_i^j(t),$$

where t_1^j is the time spent on internships in industry j, and β_i^j represents the productivity of internships in increasing industry-specific human capital for industry j.

Optimization Problem

The individual maximizes the PDV of lifetime income across different industries or firms, considering the costs of internships (industry-specific experience) and education (general knowledge):

$$\max_{t_1(t),t_2(t)} \int_0^T e^{-rt} \Big[w_0^j \left(\gamma_1 K_g(t) + \gamma_2 K_i^j(t) \right) \\ (1 - t_1(t) - t_2(t)) - p_1^j t_1(t) - p_2 t_2(t) \Big] \, dt, \qquad (1)$$

where:

- $t_1(t)$ is the time allocated to internships in industry j,
- $t_2(t)$ is the time allocated to education (general knowledge),
- w_0^j is the wage in industry j,
- p_1^j is the time cost of internships in industry j,
- p_2 is the time cost of education.

Hamiltonian

The Hamiltonian for this optimization problem is:

$$H(t, K_g, K_i^j, t_1, t_2, \lambda_g, \lambda_i^j) = e^{-rt}\left[w_0^j\left(\gamma_1 K_g + \gamma_2 K_i^j\right)(1 - t_1 - t_2) - p_1^j t_1 - p_2 t_2\right] + \lambda_g \beta_g t_2 K_g + \lambda_i^j \beta_i^j t_1 K_i^j, \quad (2)$$

where λ_g and λ_i^j are the co-state variables corresponding to general and industry-specific HC.

- λrepresents the shadow price or marginal value of an additional unit of corresponding HC.
- It indicates how much additional lifetime income can be generated from investing one more unit of effort into accumulating specific HC.

First-order conditions

For $t_1(t)$ (internship time in industry j):

$$\frac{\partial H}{\partial t_1} = -e^{-rt} w_0^j \gamma_2 K_i^j + \lambda_i^j \beta_i^j K_i^j - e^{-rt} p_1^j = 0,$$

For $t_2(t)$ (education time):

$$\frac{\partial H}{\partial t_2} = -e^{-rt} w_0^j \gamma_1 K_g + \lambda_g \beta_g K_g - e^{-rt} p_2 = 0,$$

Law of Motion for $K(t)$:

$$\dot{K}(t) = \gamma_1 \beta_g t_2(t) K_g(t) + \gamma_2 \beta_i^j t_1(t) K_i^j(t).$$

Law of Motion for $\lambda(t)$:

$$\dot{\lambda}_g(t) = -\left(e^{-rt} w_0^j \gamma_1 (1 - t_1(t) - t_2(t)) + \lambda_g \beta_g t_2(t)\right),$$

$$\dot{\lambda}_i^j(t) = -\left(e^{-rt} w_0^j \gamma_2 (1 - t_1(t) - t_2(t)) + \lambda_i^j \beta_i^j t_1(t)\right).$$

Optimal Time Allocation

Using the first-order conditions, we can determine the optimal time allocation between internships in industry j and education.
At the optimal point, the marginal benefit of spending time on internships (industry-specific) should equal the marginal benefit of spending time on education (general knowledge):

$$\frac{w_0^j \gamma_2 + p_1^j}{\beta_i^j K_i^j} = \frac{w_0^j \gamma_1 + p_2}{\beta_g K_g}.$$

This equation provides the optimal trade-off between time spent on internships and time spent on education, balancing the benefits of accumulating industry-specific experience and general knowledge.

Since the total human capital $K(t)$ is the weighted sum of $K_g(t)$ and $K_i^j(t)$, the $\dot{\lambda}$, which is negative partial derivative of H with respect to $K(t)$ can be written as:

$$-\dot{\lambda} = \frac{\partial H}{\partial K(t)} = \gamma_1 \frac{\partial H}{\partial K_g(t)} + \gamma_2 \frac{\partial H}{\partial K_i^j(t)}$$

We can solve for the relative time allocation $t_1(t)$ and $t_2(t)$, when $\dot{\lambda} = 0$:

$$\frac{t_1(t)}{t_2(t)} = \frac{\gamma_2 \beta_g}{\gamma_1 \beta_i^j}.$$

This equation shows how time should be divided between internships in industry j and education, based on their respective marginal benefits, costs, and productivity.

Intuition

- The individual will allocate time to both internships and education in a way that balances the benefits of industry-specific and general knowledge accumulation.
- The optimal time allocation depends on the relative wages in industry j, the costs of internships and education, and their productivity in increasing human capital.
- As the individual will only work in one industry at the end, they will prioritize accumulating industry-specific experience in that industry while still investing in general education to maximize lifetime earnings.

Next Steps

- External idiosyncratic shocks on industry?
- Search & Match Model, heterogeneous individual needs to find the right industry?
- TBD