33603 TA Discussion Notes

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Agenda: L1 Recap + Problem Set 1 Explanation + L2 Recap

L1: Price

Topics covered in this class:

- Macroeconomics: CIA Model, MIU Model, etc
- Frictions: staggered price, bank run, financial crisis
- Price: information of supply and demand in the market

Main tool for economic analysis: FOC

- Consumer side: marginal utility from consuming goods, marginal cost of goods
- Producer side: marginal productivity of technologies, marginal cost of labor

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- Different technologies with different productivity
- Decreasing marginal product
- Producer goal: equalize marginal productivity of different technologies. (If not, then the producer can always reallocate inputs to reach more efficient output level)

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Indifference Curve

- All points yield the same utility.
- Decreasing because two goods (leisure and consumption) are substitutes.

Price as Slope:

- Consumer: ΔConsumption/ΔLeisure: how much consumption do I want to give up in exchange for one additional unit of leisure?
- Producer: ΔProduction/ΔLabor: how much additional cost do I need to pay in exchange for one additional unit of labor?
- Wage acts as the information carrier, simplifies the resource allocation process, and reaches optimal allocation without total information (e.g., explicit production function).

Problem Set 1: Question 1

Setup

$$\begin{gathered} U(C,L) = U(C, 1-N), \quad \text{where } L + N = 1, \\ y_1 = f(A_1, N_1), \quad y_2 = g(A_2, N_2), \\ \text{Own: Given } N \text{ total, find } N_1, N_2. \end{gathered}$$

Optimization Problem

$$\max_{N_1, N_2} (y_1 + y_2) \quad \text{subject to: } N_1 + N_2 = N.$$

Lagrangian

$$\begin{aligned} \mathcal{L} &= f(A_1, N_1) + g(A_2, N_2) + \lambda(N - N_1 - N_2), \\ \frac{\partial \mathcal{L}}{\partial N_1} &= \frac{\partial \mathcal{L}}{\partial N_2} \quad \text{marginal contribution of labor to total production} \end{aligned}$$

Results

- Shadow price: λ represents the marginal contribution of constraint to total production.
- Proportional relationship: Allocation of N is proportional to marginal productivity.

Final Consumption:

$$c = y_1^* + y_2^*$$

Then calculate N according to utility maximization.

Comparision: household owns the firm v.s. does not own the firm

Production and Allocation

1. Given production $N_1, N_2 \to L$ (Leisure) & C (Consumption).
2. Allocation strategy:

(a) Households: maximize $U(C, L)$, subject to $L + N = 1$, and the budget constraint $P_C \cdot C + P_L \cdot L = W \cdot L + \text{dividend}$.

(b) Firms: maximize $\quad \pi = \text{price} \cdot \text{production} - \text{wage} \cdot \text{labor}$.

3. Market clearing:
   - Production = Consumption.
   - Labor supply = Labor demand.

Ownership Difference

	Owning	Not Owning
Agent (Maximization)	Household (Utility)	Household (Utility) + Firm (Profit)
Information	Full	market price
Budget	Production = Consumption	Income = Consumption

L2: Lucas Asset Pricing Model

P is a result of equilibrium.

Agent's Utility Maximization

Expected life-long discounted utility from consumption:

$$\max \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U(C_t), \quad 0 < \beta < 1$$

Subject to the budget constraint:

$$\text{Spending} = C_t + P_t L_t + P_t N_t \leq \text{Income (Assets + Dividends)}$$

Wealth Dynamics

$$A_{t+1} = L_t + (P_{t+1} + Y_{t+1}) \cdot N_t$$

Where:

- A_{t+1}: Wealth in the next period.
- P_{t+1}: Price at $t + 1$.
- Y_{t+1}: Dividend gain.
- N_t: Number of stocks.

Bond and Stock

Stocks: Capital gain (p_{t+1}) and dividend gain (y_{t+1}).

Bond: buy today (pay q_t), sell tomorrow (pay back \$1). Profit rate: $\frac{1}{q_t}$. In order to make bond as the riskless, 0-premium asset, the price is set as $\frac{1}{q_t} := R_t$

Value Function

$$V(A_t) = \max \left[U(C_t) + \beta \mathbb{E}_t \big(V(A_{t+1}) \big) \right]$$

- $V(A_t)$: Maximum life-long utility under state A_t.
- $U(C_t)$: Today's utility from consumption.
- $\beta \mathbb{E}_t \big(V(A_{t+1}) \big)$: Future expected discounted life-long utility.
- $V(A_t)$ is concave (inhereted from $U(C_t)$ is concave) and increasing (inhereted from $U(C_t)$ is increasing).

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Pricing Equations: The Difference

1. CAPM

For CAPM:

$$R^i = R^f + \beta_{iW}\mathbb{E}[R^W - R^f]$$

where

$$\beta_{iW} = \frac{\text{Cov}(R^i, R^W)}{\text{Var}(R^W)} = \rho\frac{\sigma_i}{\sigma_W} \quad \text{(for bivariate regression only)}$$

Therefore, we can think of a benchmark discount factor:

$$M_{t+1} = a + bR^W$$

With a discount factor, we have:

$$\mathbb{E}[M_{t+1}R^i] = 1$$

Key idea: The expectation of discounted returns shall be 0.
Given:

$$R^i = R^f + \beta_{iW}\mathbb{E}[R^W - R^f]$$

the *risk factor* is the covariance between R^i and R^W.

To be more precise: The correlation ρ and relative risk $\frac{\sigma_i}{\sigma_W}$.

2. C-CAPM

For C-CAPM:

$$\mathbb{E}[M_{t+1}R^i] = 1$$

where

$$M_{t+1} = \beta\frac{U'(C_{t+1})}{U'(C_t)}$$

For R^f and R^i, they share the equation. But for R^f, R^i is a constant.
And R^i, R^i is a random variable.

$$\mathbb{E}[M_{t+1}R^i] = \mathbb{E}[M_{t+1}]\mathbb{E}[R^i] + \text{Cov}(M_{t+1}, R^i) = \mathbb{E}\left[\beta\frac{U'(C_{t+1})}{U'(C_t)}\right]\mathbb{E}[R^i] + \text{Cov}\left(\beta\frac{U'(C_{t+1})}{U'(C_t)}, R^i\right)$$

There are two risk factors:

- Macroeconomic variables: C_t, C_{t+1}
- Covariance between R^i and consumption growth.

For risk-free assets: R^i is a constant $\Rightarrow$ covariance is 0.
For risky assets: Covariance is non-zero (usually positive) $\Rightarrow$ Risk premium.

The aggregate risk is economic cycle.

Takeaways

- Assets with high covariance with market portfolio should have high returns (CAPM).
- For C-CAPM, assets with high covariance with aggregate risk should have high returns.
- **CAPM:** Stock decreases when market goes down.
  C-CAPM: In recessions, stocks will go down, especially risky assets.

The equation:

$$\mathbb{E}[M_{t+1}R^i] = \mathbb{E}[M_{t+1}]\mathbb{E}[R^i] + \text{Cov}\left(M_{t+1}, R^i\right)$$

3. Equilibrium Asset Pricing

$$P_t = \sum_{j=0}^{\infty} \beta^j \frac{U'(C_{t+j})}{U'(C_t)} Y_{t+j}$$

Observation: We see $C_t = Y_t$ as market clears.
Thus, the stock price should equal the discounted value of future dividends.

Let:

$$M_{t+j} = \beta^j \frac{U'(C_{t+j})}{U'(C_t)}$$

$$P_t = \mathbb{E}\left[\sum_{j=0}^{\infty} M_{t+j} Y_{t+j}\right]$$

Remark: This looks similar to the DCF (Discounted Cash Flow) model indeed.

The DCF Model vs. Lucas Asset Pricing

- The DCF Model:

$$P_t = \sum_{j=0}^{\infty} \beta^j Y_{t+j}$$

- The Lucas Asset Pricing:

$$P_t = \sum_{j=0}^{\infty} \beta^j \frac{U'(C_{t+j})}{U'(C_t)} Y_{t+j}$$

Note that:

$$\beta = \frac{1}{1+R_f} \Rightarrow \beta = \frac{1}{1+\frac{1}{H}}$$

Key Differences:

- **DCF Model:** We only discount the cash value and assume that the same amount of money will have the same utility in the future.
- **Equilibrium Model:** Instead of discounting the cash value, we consider how much utility or consumption can be purchased with money in the future.

Takeaway: Discounting factors matter! Equilibrium and C-CAPM think of the *utility.*

4. Stochastic Discount Factor: Another Interpretation

Deriving the Stochastic Discount Factor

Can be derived using the Lagargarian method.

Explanation:

- **On equilibrium:** People will be indifferent between consuming or investing.
- χ_t: The shadow value of money at time t.
- We interpret $\frac{U'(C_{t+j})}{U'(C_t)} = \frac{\lambda_{t+j}}{\lambda_t}$ as how the value of money changes over time.

Remark: This shows we do not need to explicitly consider price, as it is already embedded in the model.

$\frac{\lambda_{t+j}}{\lambda_t}$ can be interpreted as price-level changes.

DCF vs. Lucas Model Revisited

- **DCF:** Assumes $\frac{\lambda_{t+j}}{\lambda_t}$ is constant $\Rightarrow$ Price level will not change.
- **Lucas Model:** Considers $\frac{\lambda_{t+j}}{\lambda_t}$ $\Rightarrow$ Accounts for changes in the value of money.

Jan 28

```
//Example of an ARMA process
//armaex.mod

//Endogenous variables
var y;
// calculated by eqs in model
// time-variant

//Exogenous variables
varexo epst;
// pre-determined
// time-invariant

//Parameters
parameters rho, theta, sd_e;

rho = 0.95;
theta = 0.05;
sd_e = 1;

//Model block
model;

y = rho*y(-1) + epst + theta*epst(-1);

end;

initval;
```

```
y = 0;
end;

steady;
check;

// shock block
shocks;
var epst;
stderr sd_e;
// std error
end;
// epst: mean 0, var 1, white noise process

stoch_simul(periods=300,irf=40,order=1);
//stoch_simul(periods=300,irf=40);
// period=300: a time-series sequences of 300 units
// irf=40: use 40 units to calculate IRF
// IRF: dynamic response of y to epst
// order=1: polynomial order 1, use first-order approximation to solve

//Note: The default is to solve the problem with a 2nd-order linearization.
//However, as the system is linear, it is good in this case to set order=1.

figure;
plot(oo_.endo_simul(strmatch('y', M_.endo_names, 'exact'), :));
// plot time series
title('$y$','Interpreter','latex');
```

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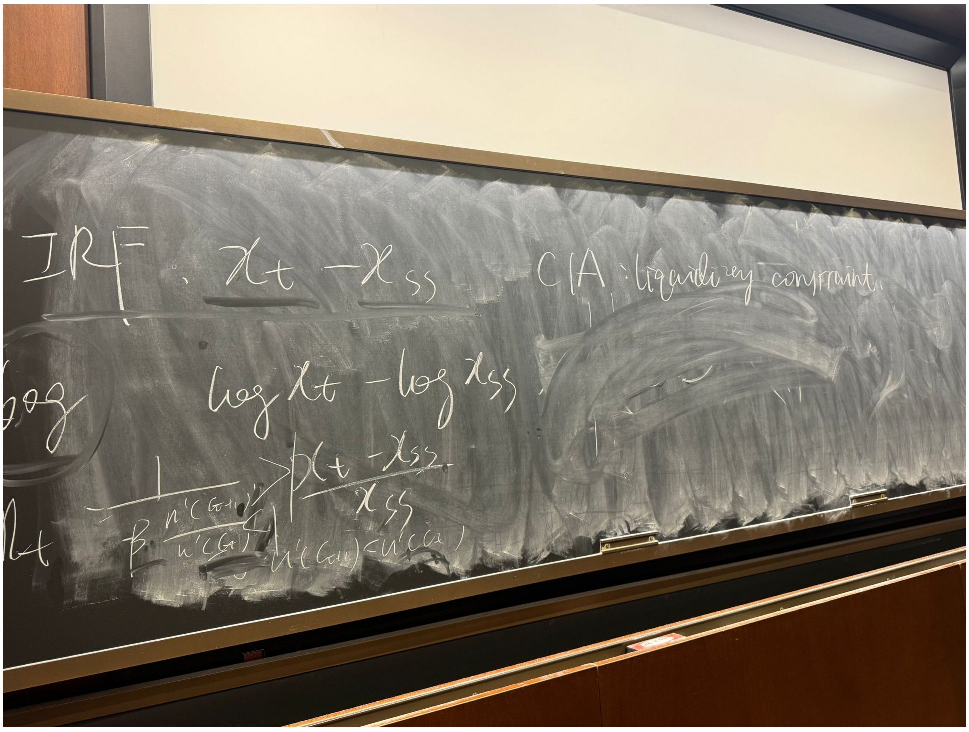
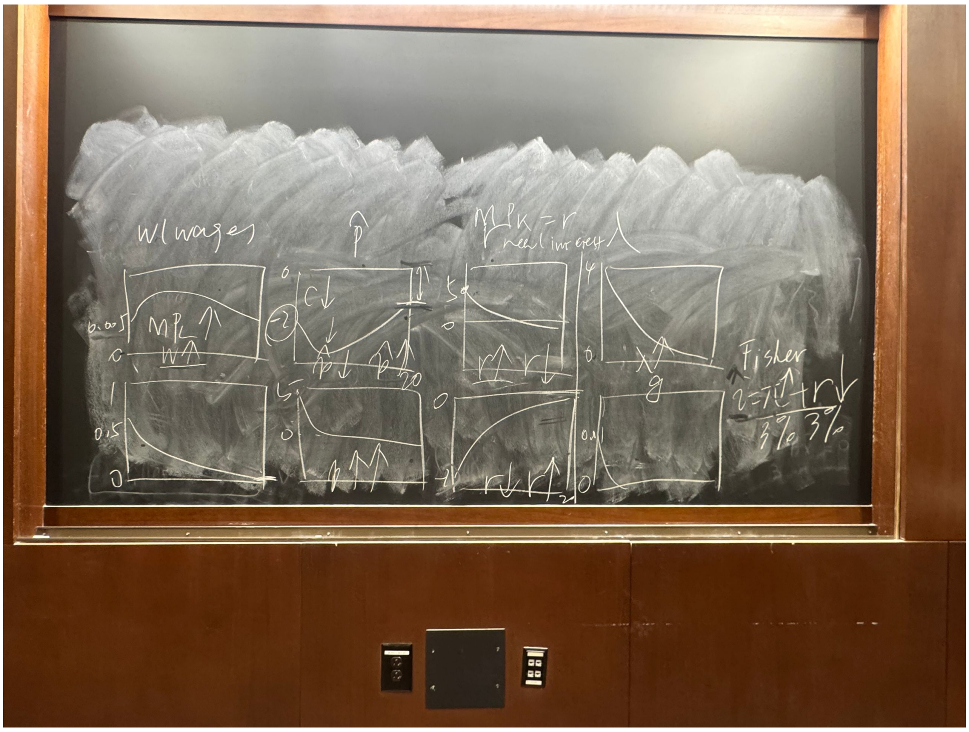

IRF : x_t − x_ss
CIA : liquidity constraint
log x_t − log x_ss


cons.
prod. shock
money



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Slides01Information

Introduction to the core role of prices in economic activities, and explores how to use the price system to coordinate scattered information and achieve the best allocation of resources.

1. Reasons for the existence of prices: not only as a medium of exchange, but also as an information transmission mechanism, through which the scattered knowledge owned by various economic entities is integrated to achieve the best use of resources.
2. Role of price mechanism: resource changes in the market are reflected through the price mechanism, thereby guiding economic entities to adjust production and consumption.
3. Rational economic order: the marginal substitution rate between products should be equal, ensures the efficiency of resource allocation. However, due to the dispersion and incompleteness of information, it is difficult to directly calculate this optimal condition.
4. Labor allocation under 2 different technologies, with price and no price scenarios:
   - No price situation: It is necessary to know the specific information and welfare function of each technology to determine the optimal labor allocation.
   - With price situation: Each industry only needs to decide the labor input according to the market price, and finally achieve the optimal resource allocation and welfare maximization of the overall economy.
5. Function of price
   - Price is not only a simple transaction medium, but also a key tool for transmitting decentralized information in the economy, helping each market participant to make independent and effective decisions.
   - Although the optimal allocation can be determined by mathematical methods (equal marginal substitution rate) in theory, relying on price signals can achieve this goal more efficiently in practice, avoiding the inconvenience caused by information concentration.

- The price system allows each department or enterprise to make the best choice based on its own local information, and finally spontaneously achieve the global optimal configuration, which is one of the important reasons why the market economy is better than the centralized planned economy.

Slides02AssetPricing

Introduction to asset pricing models, derivations and limitations.

1. Models:
   - Consumption capital asset pricing model (CCAPM): uses the random discount factor $M_{t+1} = \beta\frac{u'(c_{t+1})}{u'(c_t)}$ to obtain the pricing relationship $E_t[M_{t+1}R_i] = 1$, explains the connection between asset returns and consumption growth risk.
   - Equilibrium asset pricing and Lucas Tree: uses the bubble-free condition in the market to determine asset prices in an equilibrium framework where only trees (or "tree models") exist, that is, asset prices are equal to the infinite sum of the discounted values of future dividends (or consumption)
   - Mehra and Prescott (1985): constructs variables $v_k = p(\sigma_k)u'(\sigma_k)$ and uses the linear system $v = \alpha + \beta Pv$ to solve, thereby obtaining the calculation method of price and risk premium.
2. Why stochastic discount factor and consumption risk are important? Asset prices are determined by the discounted value of future dividends (or consumption), and the discount factor reflects the uncertainty and risk of consumption growth. The consumption CAPM model emphasizes that consumption risk is a key factor affecting asset returns.
3. Condition for equilibrium pricing: In an equilibrium market, asset prices must meet the no-bubble condition, that is, the limit of the future discount term is zero, which ensures that the price fully reflects the fundamental information, and thus derives the infinite discount and relationship between the price and future dividends.
4. Use the asset pricing models to interpretate risk premium: Numerical simulations show that only under relatively high risk aversion coefficients can the model produce risk premiums and Sharpe ratios that are consistent with the actual market, which also reveals the limitations of the classical model in explaining asset price fluctuations.

Slides03TermStructure

Introduction to the construction of yield curve and its relationship with macroeconomics.

1. Construction of the Yield Curve
   - Definition of yield: The constant annual interest rate implied by a bond's quoted price (yield to maturity or spot rate).
   - Definition of yield curve: A graph plotting yields against maturities. A horizontal curve would imply identical income options across maturities, but typically, the yield curve is upward sloping.
   - Calculation: The presentation illustrates bond pricing using zero-coupon bonds (e.g., pricing a 1-year bond versus a 2-year bond to derive yields like 5% for one year and 6% for two years).
   - Economic interpretation: It explains how bond prices adjust through market forces until returns for different maturities become equivalent.
2. Short rates and yield: Short rates are the interest rates prevailing over one period, while yields to maturity reflect the average return over the entire term of the bond.

3. Forward rates and yield: Forward rates are derived from yields of bonds with different maturities using relationships like
$$(1 + y_n)^n = (1 + y_{n-1})^{n-1}(1 + f_n)$$
indicating the implied future short rates. It incorporates both expectations of future short-term rates and risk premiums due to uncertainty. Because future short rates (like r_2) are uncertain, the forward rate often deviates from the simple expected short rate.

4. Uncertainty, risk premium and yield: Investors require a risk premium for bearing this uncertainty, this is reflected in the observed upward slope of the yield curve when short-term investors dominate.

5. Terrm structure:

   - Expectations Hypothesis: forward rates equal the market's expectation of future short rates.
   - Liquidity Preference (or Term Premium) Theory: investors demand additional compensation (a liquidity premium) for holding longer-term bonds, which are riskier due to uncertainty.

6. Relationship with economy: The slope of the yield curve (e.g., the spread between long-term and short-term rates) has been empirically linked to future economic indicators such as GDP, consumption, and investment. An indicator such as the spread between the 10-year Treasury rate and the 3-month T-bill rate is used in practice to forecast economic conditions.

Slides04AssetPricingHJB

Introduction to the critical benchmark for asset pricing models: the Hansen-Jagannathan bounds, which serve as a necessary condition for any model of the SDF.

1. Hansen-Jagannathan (H-J) Bounds

   - Hansen and Jagannathan (1991) developed a method to derive a frontier (bounds) for the possible mean, and standard deviation of any stochastic discount factor (SDF) that is consistent with observed asset returns. This "H-J bound" provides a benchmark for evaluating candidate SDFs.
   - Market Price of Risk: The SDF, m_{t+1}, enters the Euler equation:
     $$q_t = E_t\left[m_{t+1} p_{t+1}\right]$$
     By rearranging and considering the covariance between m_{t+1} and asset payoffs, one can derive that the expected excess return on an asset is bounded by a term proportional to the market price of risk, defined as:
     $$\frac{\sigma_t(m_{t+1})}{E_t(m_{t+1})}$$
   - Linear form of the SDF:
     $$y = a + x'b + e$$
     where e is orthogonal to x. This decomposition shows that the variance of y must be at least as large as that of its predictable part, which sets the lower bound.
   - If the risk (volatility) of the SDF increases by one unit, the expected return on a risky asset must rise by the market price of risk.
   - The bound implies that for any candidate SDF to be consistent with the observed asset returns, its volatility (relative to its mean) must be high enough to account for the excess returns seen in the market.

2. CRRA SDF

   - A common candidate SDF is derived from a CRRA utility function:
     $$m_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$$

- Assumption: log consumption follows a random walk and how it implies that $\log m_{t+1}$ is normally distributed.

3. Challenge for reconciling theory with empirical observations: If a candidate SDF does not generate enough volatility relative to its mean to match the observed excess returns, then the model is misspecified or requires unrealistically high risk aversion.

Slides05CAPMContraints

Introduction to the portfolio optimization problem with N risky assets, effects and implications of short sale constraint.

1. Optimization problem
   - The optimization problem is to minimize portfolio variance subject to two key constraints: expected return constraint $\omega'\mu = \mu_p$, and full investment constraint $\omega'\mathbf{1} = 1$.
   - Using Lagrange multipliers, an analytical solution is derived. The optimal portfolio weights are expressed in the form:
   $$\omega_p^* = g + h\,\mu_p,$$
   where the vectors g and h are functions of the inverse covariance matrix and the mean returns.
   - The resulting minimum-variance frontier is shown to be a quadratic (parabolic/hyperbolic) function in the risk-return (mean-variance) space.
2. Short-Sales Constraints
   - In practice, fund managers often face constraints such as prohibitions on short sales (e.g., $\omega_i \geq 0$ for all assets) or limits on maximum allocations.
   - Effect of imposing constraints: When short-sales constraints are imposed, the optimal portfolio weights change significantly.
   - Results: attainable minimum-variance frontier shifts, and cconstrained portfolios may have higher risk (variance) for the same level of expected return compared to their unconstrained counterparts.
   - Practical implications for asset allocation: The changes in optimal weights due to constraints can affect both risk management and expected return outcomes.

Slides06MoneyCIA

Introduction to the Cash-in-Advance (CIA) Model, and welfare cost of inflation.

1. Without frictions, money would not be used because agents could trade directly. The CIA constraint forces agents to hold money from previous periods to purchase consumption, thereby giving money a role in the economy.
2. Household Framework & Preferences
   - The household is split into two roles: a worker (who produces goods and brings money into the household) and a shopper (who uses money held from the previous period to buy consumption).
   - The representative household maximizes utility given by
   $$E_0 \sum_{t=0}^{\infty} \beta^t\, u(c_t, h_t),$$
   where a typical specification is $u(c_t, h_t) = \log c_t + B\,h_t$. Here, B is chosen such that leisure (or equivalently, lower labor hours) has a positive value, reflecting the fact that work is costly in utility terms.

3. Production and Money Supply

   - Output is produced via a standard Cobb-Douglas function,

   $$y_t = \lambda_t K_t^{\theta} H_t^{1-\theta},$$

   with total factor productivity λ_t following a stochastic process.

   - Money enters the model through a CIA constraint. Households carry over money m_{t-1} from the previous period, while the money supply M_t grows at a rate g_t, which is also stochastic. The government finances its operations by transferring the net increase in money $(M_t - M_{t-1})$ to households.

4. *Constraints and Normalization

   - The key constraint is that consumption must be financed by money held from the previous period and any new money created:

   $$p_t c_t \leq m_{t-1} + (g_t - 1)M_{t-1}.$$

   - Flow Budget Constraint: sum of consumption, investment, and money holdings $\leq$ sum of labor income, capital income, and money transfers
   - Normalizing nominal variables by the money supply is essential for obtaining a stationary equilibrium. To deal with the nonstationarity induced by money growth, nominal variables (e.g., prices and money) are normalized by the money supply M_t, so that the normalized money holding $\hat{m}_t = m_t/M_t$ is set to one in equilibrium.

5. The model's optimality conditions are derived using a Lagrangian approach. These conditions generate a system of equations that capture the trade-offs between consumption, labor, investment, and money holding.

6. Welfare Cost of Inflation: An increase in inflation erodes the real value of money, leading households to substitute away from consumption goods that require money. This substitution reduces labor incentives and lowers labor supply, increasing leisure consumption.

Slides07MoneyMIU

1. Money in Utility (MIU) approach incorporates money directly into the utility function. This formulation gives money intrinsic value for holding it, making it an integral part of agents' decisions. Also, by controlling the degree of substitution between money and consumption through parameterization, the flexibility simplifies the explanation for why agents hold money even in the absence of explicit transaction costs.

2. Preferences

   - The representative agent maximizes:

   $$E_0 \sum_{t=0}^{\infty} \beta^t \, u(c_t, \frac{m_t}{P_t}, h_t)$$

   where the utility function is specified as:

   $$u(c_t, \frac{m_t}{P_t}, h_t) = \log c_t + D \, \log\left(\frac{m_t}{P_t}\right) + B \, h_t.$$

   Here, D governs the utility derived from holding real money balances, and B reflects the disutility of labor. The parameter D allows policymakers and researchers to control the degree of substitution between real money balances and consumption, thereby influencing the responsiveness of money demand to changes in economic conditions.

- The household faces a budget constraint:

$$c_t + \frac{m_t}{P_t} + k_{t+1} = w_t\, h_t + r_t\, k_t + (1-\delta)k_t + \frac{(g_t - 1)M_{t-1}}{P_t},$$

where money carried over from the previous period (and any new money created) finances consumption.

3. Production & Money Supply

- Production Function:

$$y_t = \lambda_t\, K_t^{\theta} H_t^{1-\theta},$$

with total factor productivity (TFP) λ_t following a stochastic process.

- Money: Money is carried over from period to period. The money supply grows at a rate $g_t = M_t/M_{t-1}$, and its evolution is given by:

$$\log g_{t+1} = (1-\pi)\log \bar{g} + \pi \log g_t + \epsilon_{g,t+1}.$$

The government transfers the net increase in money to households.

4. Equilibrium Conditions

- The first-order conditions from the optimization problem—together with market clearing and production conditions—yield a system of equations in endogenous variables.
- The steady state is characterized by constant values for consumption, labor, capital, output, wages, and interest rates. Key steady-state equations are provided (e.g., $r = 1/\beta - (1-\delta)$, $c = -w/B$, etc.).
- To analyze dynamics, the model is log-linearized ($\tilde{x} = \log x$), making it easier to study the responses of variables to shocks.

5. Results: The MIU model typically yields smaller real effects from monetary shocks compared to the CIA model because the utility benefit from holding money offsets some of the adjustment mechanisms.

Slides08StaggeredPrices

Introduction to economic rationale behind staggered (Calvo) pricing and why nominal rigidities are essential for monetary non-neutrality.

1. Nominal Rigidity & Real Effects

- Money is a nominal variable. If prices were fully flexible and adjusted instantly, changes in the money supply would only affect nominal variables (like the price level) and leave real variables unchanged.
- To generate real effects of monetary shocks, the model imposes a friction on price adjustments by assuming that firms set prices in a staggered (or Calvo) fashion.

2. Calvo pricing framework

- Each firm, indexed by $k \in [0,1]$, produces an intermediary good. In each period, a firm has a probability $1-\rho$ of being able to reset its price optimally and a probability ρ of keeping its price unchanged from the previous period. The higher ρ is, the stickier the prices are, which magnifies the real effects of monetary policy.
- Intermediary firms produce goods with a CES production function, and final-goods producers aggregate these goods using a CES aggregator.

- The demand for an individual firm's output is given by a function of the ratio of the aggregate price level P_t to the firm's own price $P_t(k)$, raised to a power determined by the elasticity of substitution ψ:

$$Y_t(k) = Y_t \left(\frac{P_t}{P_t(k)} \right)^{\psi}.$$

- The aggregate price level is determined by the formula:

$$P_t = \left[\int_0^1 P_t(k)^{1-\psi}\, dk \right]^{\frac{1}{1-\psi}}.$$

- Firms that are allowed to adjust their prices choose an optimal reset price $P_t^*(k)$ that maximizes their expected discounted profits over the periods during which the price remains effective, incorporating the probability ρ of not being able to change the price in future periods.
- The optimal reset price can be expressed as a markup over expected future marginal costs.

3. Households and Market Equilibrium

   - Households purchase the final good at the aggregate price P_t and supply labor. Their decisions, together with those of firms, help determine equilibrium in both goods and labor markets.
   - The full model comprises the behavior of intermediary firms, the aggregation of their prices into a final price level, and household optimization.

4. Log-linearization: around the steady state, where new variables are defined as the log deviations from steady state. By log-linearizing the aggregate price-setting equation, one obtains an expression that relates current inflation to expected future inflation and real marginal cost (or, equivalently, the output gap).

5. Results: Simulations show that when ρ is high (i.e., prices are very sticky), monetary policy has significant real effects, leading to pronounced responses in output, employment, and inflation. When ρ is very low (nearly flexible prices), the model converges to monetary neutrality.

6. Real effects of monetary policy: Staggered price setting introduces nominal rigidities that cause prices to adjust slowly. This delay allows changes in the money supply (or nominal shocks) to affect real variables like output and employment.

7. Relationship with Phillips curve: The model naturally yields a Phillips curve where inflation is determined by expected future inflation and the real marginal cost. The slope of the Phillips curve depends on the degree of price stickiness (captured by ρ) and other structural parameters.

8. Policy implications: The effectiveness of monetary policy depends critically on the extent of price rigidity. With high price stickiness, monetary interventions can have strong real effects, while with low stickiness, the economy behaves almost as if prices were fully flexible.

Slides09WelfareCostInflation

Introduction to combining individual-level decisions on trade frequency, and extension from discrute time to continuous time setting.

1. Intuitions

   - This model gives agents the flexibility to choose when to trade bonds for money, allowing for continuous time trading. Trading is modeled in continuous time via the differential equation:

   $$\dot{M}(t) = -P(t)c(t)$$

   so that money holdings decline over a "holding period" until a new trade occurs. During holding periods $[T_j, T_{j+1})$, agents begin with a high money balance right after trading and then see it decline as they spend.

- Transitioning from discrete to continuous time provides greater flexibility in capturing the timing of financial trades. The use of integration and differential equations allows the model to capture the gradual decline of money holdings over a holding period.

2. Heterogeneous agents

   - Agents are differentiated by their position within a holding period (indexed by $n \in [0, N)$), reflecting differences in their money and bond holdings. Market clearing is achieved by integrating individual money and bond holdings across the continuum of agents.
   - Portfolio Frictions & Costs: Trading between bonds and money is not costless. There are frictions (denoted by a cost parameter, Γ) that make frequent rebalancing costly. These costs force agents to optimally choose the length $N = T_{j+1} - T_j$ of their holding period.
   - Optimality condition:
   $$N \approx \sqrt{\frac{2\gamma}{r}}$$
   where γ represents the proportional trading cost and r is the nominal interest rate. Clearly, trading costs (parameter γ) and nominal interest rates influence the frequency of trades.
   - Arrgegate money demand: By aggregating over all agents (each at different points in their holding period), the model derives an aggregate money demand function (expressed as a money-income ratio $m(r)$) that is more elastic than in standard CIA models. This elasticity captures how higher inflation (and thus higher nominal rates) force agents to rebalance more frequently, leading to a reduction in the demand for liquid money.

3. Welfare Cost of Inflation: Inflation imposes a welfare cost because it forces agents to incur additional costs from increased trading frequency. With higher inflation, the nominal interest rate rises, making it more attractive to convert bonds into money repeatedly to minimize losses from holding depreciating cash.

4. Result comparision: When trading frequency and associated portfolio rebalancing costs are taken into account, the welfare cost of inflation is substantially higher than that predicted by standard CIA models.

5. Policy implications: The results imply that even moderate inflation can generate significant inefficiencies and welfare losses due to the distortions in financial trading behavior, provides an important rationale for low and stable inflation policies.

Slides10OTC

Introduction to an extended framework of CIA, with Policy implications of the welfare cost of inflation.

1. Intuitons

   - Extended CIA model by incorporating decisions on capital, labor, and distortionary taxation, with the purposes to analyze two financing methods for increased government expenditures: financing via higher inflation vs. using taxes.
   - This extension allows the frequency of financial trades (i.e., the holding period for money) to be endogenous.
   - Welfare costs are defined as the income compensation (or loss) required to leave agents indifferent between two financing policies (e.g., financing with inflation versus with taxes).

2. Results

   - Allowing agents to choose their trading frequency (i.e., making it endogenous) reveals that inflation has a much larger welfare cost than predicted by traditional CIA models.
   - As inflation increases, agents choose to trade more frequently (to minimize the opportunity cost of holding depreciating cash), thereby expanding the financial sector. This greater rebalancing frequency leads to a more elastic demand for money and significantly alters welfare cost calculations.

- The model predicts that using inflation to finance government spending increases the cost (in terms of income compensation required) by roughly 1 percentage point more than when using taxes.

3. Policy implications: Contrary to some CIA-based analyses where moderate inflation might seem optimal, in an environment where the financial sector reacts to inflation (with increased trading frequency), it is optimal to finance higher government expenditures with taxes rather than inflation. Policies aimed at keeping inflation low may yield significant welfare gains.

4. Financial sector dynamics: The enlargement of the financial sector (through more frequent bond-to-money trades) due to higher inflation amplifies the distortions in the economy. Seigniorage revenues and their relationship to money demand play a central role in the welfare analysis.

Slides11OTCCapital

Introduction to OTC markets, characterized by decentralized and bilateral trading with search frictions, incorporating the trading of capital assets.

1. Intuition: Unlike centralized exchanges, OTC markets require investors to search for dealers with whom they can negotiate trades, introducing additional frictions that affect asset prices and allocation.

2. Environment

   - continuous time and features two types of agents: investors and dealers. Both agents have linear utility functions $u(x, h) = x - h$, where x denotes consumption and h represents hours of work.
   - Investors hold capital and face productivity shocks that determine their productivity type. They are not able to trade in a competitive market directly, but adjust their capital holdings by trading with dealers in an OTC market.
   - Dealers do not produce the consumption good or hold capital; they facilitate trades by offering intermediation services in a competitive setting. They earn an intermediation fee, but they hold no inventory.

3. Technology and production

   - Two production technologies exist: one that uses labor (where one unit of labor produces one unit of the good) and another that uses capital. Investors face productivity shocks when using capital. Each investor draws a productivity type j from a set $\{1, \ldots, I\}$ with probabilities π_j, and these shocks arrive at a Poisson rate δ.
   - Investors meet dealers through a random matching process with a Poisson arrival rate σ. In the limit as $\sigma \to \infty$, search frictions vanish and the market becomes competitive.
   - When an investor meets a dealer, they negotiate a trade agreement that specifies a new capital position k' and a fee ϕ paid to the dealer. The agreement is derived using Nash bargaining. The dealer's bargaining power is captured by a parameter θ, with the solution given by:

     $$\phi_i(k) = \theta\Big[V_i(k^i) - V_i(k) - p\big(k^i - k\big)\Big],$$

     where $V_i(k)$ is the investor's value function for productivity type i and current capital k, k^i is the optimal post-trade capital, and p is the price per unit of capital adjustment.

4. Value functions

   - The investor's value function $V_i(k)$ satisfies a Bellman equation that includes: flow benefits (dividends or production benefits) from holding capital, adjustment costs incurred during trades with dealers (captured by the fee and the price impact), and transition terms due to random changes in productivity. Investors choose the optimal capital adjustment k' to maximize their net value $V_i(k') - pk'$. The intermediation fee extracted by the dealer depends on the gains from trade and the dealer's bargaining power.

- Dealers' value function is derived similarly, reflecting their role in collecting fees from trades.

5. Equilibrium and Trade Dynamics

   - An equilibrium is characterized by a set of prices, trade agreements, and value functions such that: Investors' Bellman equations hold, dealers earn zero profit (competitive condition), the negotiated trade (via Nash bargaining) maximizes the joint surplus split according to bargaining power.
   - The model yields a unique steady state distribution of investors by productivity type and capital holdings. Under steady state, the flow into each state equals the flow out, and as search frictions decrease (i.e. $\sigma \to \infty$), the distribution converges to the competitive benchmark. As the search intensity increases (i.e., frictions decline), the equilibrium converges to the efficient, competitive outcome.

6. Results: Search and bargaining frictions in OTC markets lead to noncompetitive prices for capital assets, with an intermediation fee that reflects the cost of search. These frictions affect the optimal capital adjustment decisions of investors and the overall distribution of capital holdings. As search frictions vanish, the OTC market outcome converges to the frictionless, competitive case (with zero bid–ask spreads and fees).

7. Bargaining results: The Nash bargaining solution determines how much surplus is captured by dealers versus investors. Higher dealer bargaining power (a larger θ) results in higher fees, reducing investors' net gains.

8. Policy implications: The model highlights the importance of facilitating better search mechanisms and increasing market transparency.